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Reg. No. : ........................

Name : ...............................

# Fourth Semester B.Tech. Degree Examination, May 2014
## (2008 Scheme)
## 08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)

Time: 3 Hours Max. Marks: 100

***Instructions :*** *Answer **all** questions from Part – **A** each question carries 4 marks and **one full** question from **each** module of Part – **B** each **full** question carries **20** marks.*



PART – A

(10×4=40 Marks)

1. Prove that $f(z) = e^z$ is differentiable everywhere and find its derivative.

2. Prove that an analytic function with constant argument is constant.

3. Choose 'a' so that the function $u = x^3 + axy^2$ is harmonic, find its harmonic conjugate.

4. Prove that a bilinear transformation preserves cross ratio.

5. Evaluate $\int_0^{1+i} (x^2 - iy)\, dz$ along $y = x$ and $y = x^2$. Are they equal ?

6. Using Cauchy's integral formula, evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is $|z| = 2$.

7. Obtain the Taylor series expansion of $f(z) = \frac{1}{z^2}$ about $z = 2$.

8. Explain Newton-Raphson method.

9. Apply Lagrange's interpolation formula to find f(6) given f(1) = 2, f(2) = 4, f(3) = 8, f(4) = 16 and f(7) = 128.

10. The following table gives the values of f(x) at equal intervals of x.

| x : | 0 | 0.5 | 1 | 1.5 | 2 | 2.5 | 3 |
|---|---|---|---|---|---|---|---|
| **f(x) :** | 0 | 0.7071 | 1 | 1.2247 | 1.4142 | 1.5811 | 1.732 |

Evaluate $\int_0^3 f(x)\,dx$ by

i) Simpson's $\frac{1}{3}$ rule

ii) Trapezoidal rule. **(10×4=40 Marks)**

## PART – B

### Module – I

11. a) Show that $f(z) = \dfrac{xy^2(x+iy)}{x^2+y^4}$ for $z \neq 0$

$= 0$ for $z = 0$

is not differentiable at z = 0.

b) If f(z) = u + iv is analytic, prove that the families of curves $u = c_1$ and $v = c_2$ where $c_1$ and $c_2$ are constant cut orthogonally.

c) Find the bilinear transformation that maps the points z = 1, i, –1 onto w = i, 0, – i. Hence find the image of | z | < 1.

12. a) Prove that f(z) = xy + iy is everywhere continuous but not analytic.

b) Given $u + v = \dfrac{2\sin 2x}{e^{2y} + e^{-2y} - 2\cos 2x}$ find f(z) = u + iv by Milne-Thompson method.

c) Determine the region of the w-plane into which the triangular region bounded by x = 1, y = 1 and x + y = 1 is mapped by $w = z^2$.

## Module – II

13. a) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2 (z-2)} dz$ where C is $|z| = 2$.

b) Obtain the Laurent's series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in $0 < |z-1| < 1$ and hence find its residue about $z = 1$.

c) Evaluate $\int_0^{\infty} \frac{1}{(a^2 + x^2)^2} dx$ using Cauchy's residue theorem.

14. a) Determine the nature and singularities of

i) $\frac{e^{2z}}{(z-1)^4}$ ii) $ze^{\frac{1}{z^2}}$.



b) Evaluate $\int_{|z|=3} \frac{e^z}{(z+2)(z+1)^2} dz$ by Cauchy's residue theorem.

c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4\cos\theta} d\theta$.

## Module – III

15. a) Find the approximate value correct to three places of decimals of the real root which lies between –2 and –3 of the equation $x^3 - 3x + 4 = 0$ by regula falsi method.

b) Solve by Gauss-Seidal method given

$10x - 2y - z - 10 = 3$

$-2x + 10y - z - w = 15$

$-x - y + 10z - 2w = 27$

$-x - y - 2z + 10w = -9$.

c) From the following table :

| | | | | | | |
|---|---|---|---|---|---|---|
| **x :** | 0.1 | 0.2 | 0.3 | 0.4 | 0.5 | 0.6 |
| **f(x) :** | 2.68 | 3.04 | 3.38 | 3.68 | 3.96 | 4.21 |

Find f(0.7) by appropriate interpolation formula.

16. a) Find the real root of $x^3 - 2x = 5$ correct to three decimal places by bisection method.

b) Find the value of y when x = 0.1, 0.2 and 0.3 by Runge-Kutta method given $y' = xy + y^2$; $y(0) = 1$.

c) Using Taylor series method solve $y' = x^2 - y$; $y(0) = 1$ at x = 0.1, 0.2 and 0.3.

**(3×20=60 Marks)**