



Reg. No. :

Name :

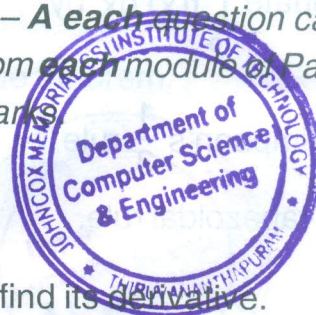
Fourth Semester B.Tech. Degree Examination, May 2014
(2008 Scheme)

08.401 : ENGINEERING MATHEMATICS – III (CMPUNERFHB)

Time: 3 Hours

Max. Marks: 100

Instructions: Answer **all** questions from Part – A **each** question carries 4 marks and **one full** question from **each** module of Part – B **each full** question carries 20 marks.



PART – A

1. Prove that $f(z) = e^z$ is differentiable everywhere and find its derivative.
2. Prove that an analytic function with constant argument is constant.
3. Choose 'a' so that the function $u = x^3 + axy^2$ is harmonic, find its harmonic conjugate.
4. Prove that a bilinear transformation preserves cross ratio.
5. Evaluate $\int_0^{1+i} (x^2 - iy) dz$ along $y = x$ and $y = x^2$. Are they equal ?
6. Using Cauchy's integral formula, evaluate $\int_C \frac{e^{2z}}{(z+1)^4} dz$ where C is $|z| = 2$.
7. Obtain the Taylor series expansion of $f(z) = \frac{1}{z^2}$ about $z = 2$.
8. Explain Newton-Raphson method.



9. Apply Lagrange's interpolation formula to find $f(6)$ given $f(1) = 2$, $f(2) = 4$, $f(3) = 8$, $f(4) = 16$ and $f(7) = 128$.
10. The following table gives the values of $f(x)$ at equal intervals of x .

$x :$	0	0.5	1	1.5	2	2.5	3
$f(x) :$	0	0.7071	1	1.2247	1.4142	1.5811	1.732

Evaluate $\int_0^3 f(x) dx$ by

- Simpson's $\frac{1}{3}$ rule.
- Trapezoidal rule.

(10×4=40 Marks)

PART - B
Module - I

11. a) Show that $f(z) = \frac{xy^2(x + iy)}{x^2 + y^4}$ for $z \neq 0$
 $= 0$ for $z = 0$

is not differentiable at $z = 0$.

- b) If $f(z) = u + iv$ is analytic, prove that the families of curves $u = c_1$ and $v = c_2$ where c_1 and c_2 are constant cut orthogonally.
- c) Find the bilinear transformation that maps the points $z = 1, i, -1$ onto $w = i, 0, -i$. Hence find the image of $|z| < 1$.
12. a) Prove that $f(z) = xy + iy$ is everywhere continuous but not analytic.

b) Given $u + v = \frac{2 \sin 2x}{e^{2y} + e^{-2y} - 2 \cos 2x}$ find $f(z) = u + iv$ by Milne-Thompson method.

- c) Determine the region of the w -plane into which the triangular region bounded by $x = 1$, $y = 1$ and $x + y = 1$ is mapped by $w = z^2$.



Module – II

13. a) Evaluate $\int_C \frac{\sin \pi z^2 + \cos \pi z^2}{(z-1)^2(z-2)} dz$ where C is $|z| = 2$.
- b) Obtain the Laurent's series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in $0 < |z-1| < 1$ and hence find its residue about $z = 1$.
- c) Evaluate $\int_0^\infty \frac{1}{(a^2 + x^2)^2} dx$ using Cauchy's residue theorem.

14. a) Determine the nature and singularities of

i) $\frac{e^{2z}}{(z-1)^4}$

ii) $ze^{\frac{1}{z^2}}$



- b) Evaluate $\int_{|z|=3} \frac{e^z}{(z+2)(z+1)^2} dz$ by Cauchy's residue theorem.

c) Evaluate $\int_0^{2\pi} \frac{\cos 2\theta}{5 + 4 \cos \theta} d\theta$.

Module – III

15. a) Find the approximate value correct to three places of decimals of the real root which lies between -2 and -3 of the equation $x^3 - 3x + 4 = 0$ by regula falsi method.

- b) Solve by Gauss-Seidal method given

$$10x - 2y - z - 10 = 3$$

$$-2x + 10y - z - w = 15$$

$$-x - y + 10z - 2w = 27$$

$$-x - y - 2z + 10w = -9.$$



c) From the following table :

$x :$	0.1	0.2	0.3	0.4	0.5	0.6
$f(x) :$	2.68	3.04	3.38	3.68	3.96	4.21

Find $f(0.7)$ by appropriate interpolation formula.

16. a) Find the real root of $x^3 - 2x = 5$ correct to three decimal places by bisection method.

b) Find the value of y when $x = 0.1, 0.2$ and 0.3 by Runge-Kutta method given $y' = xy + y^2; y(0) = 1$.

c) Using Taylor series method solve $y' = x^2 - y; y(0) = 1$ at $x = 0.1, 0.2$ and 0.3 .

(3×20=60 Marks)
